

**QUESTION 2009**

**Group - A**  
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following:

i) The value of  $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$  is

- a) 1       b) 4      c) 0      d) 2

ii) The value of  $\int_1^2 dx$  is equal to

- a) 1      b) 2      c) 3      d) 0

iii) If  $x = -1$  is a root of the equation  $x^2 - x - k = 0$ , then the value of  $k$  is

- a) 1      b) 0      c)  $\sqrt{2}$        d) 2

iv) If  $\alpha, \beta, \gamma$  be the roots of the equation  $x^3 - 3x^2 + 6x - 2 = 0$ , then  $\alpha + \beta + \gamma$  is

- a) 2      b) 1       c) 3      d) none of these

v) If  $A = \{1, 2, 3, 4\}$  and  $B = \{2, 4, 6\}$ , then  $A \Delta B$  is

- a)  $\{1, 2\}$       b)  $\{1, 2, 3, 6\}$        c)  $\{1, 3, 6\}$       d)  $\{6\}$

vi) What is the order of the matrix  $B$ , if  $[3 \ 4 \ 2] B = [2 \ 10 \ 3 \ 6 \ 9]$ ?

- a)  $1 \times 5$        b)  $1 \times 3$       c)  $3 \times 5$       d)  $5 \times 3$

vii) The degree of the polynomial  $f(x) = (x^2 + x - 2)/(x - 1)$  is

- a) 0       b) 1      c) 2      d) 3

viii) If  $y = \log x^2$ , the value of  $\frac{d^2 y}{dx^2}$  is

- a)  $\frac{2}{x^3}$        b)  $-\frac{2}{x^2}$       c)  $\frac{2}{x}$       d)  $2x$

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ix) The value of  $t$  for which the matrix  $\begin{bmatrix} 2 & 0 & 1 \\ 5 & t & 3 \\ 0 & 3 & 1 \end{bmatrix}$  is singular, is

a)  $-\frac{3}{2}$

b) 2

c)  $\frac{3}{2}$

d) -2

x)  $\lim_{x \rightarrow 0} (1+x)^{1/x}$  is equal to

a) 1

b)  $e$

c)  $\infty$

d) 0

xi) If  $\alpha, \beta, \gamma$  be the roots of the equation  $x^3 - 3x^2 + 6x - 2 = 0$ , then  $\sum \alpha\beta$  is

a) 3

b) 6

c) 2

d) none of these

xii) If  $A = \{1, 2, 3\}$  and  $B = \{2, 3, 6\}$ , then  $A \cup B$  is

a)  $\{1, 2, 3\}$

b)  $\{2, 3\}$

c)  $\{1, 2, 3, 6\}$

d) none of these

**Group - B**

**(Short Answer Type Questions)**

2. Evaluate the integral  $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$ .

See Topic: INDEFINITE INTEGRATIONS, Short Answer Type Question No. 10.

3. If  $u = \log r$  and  $r^2 = x^2 + y^2 + z^2$ , prove that  $r^2 \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right] = 1$ .

See Topic: SUCCESSIVE DIFFERENTIATION, Long Answer Type Question No. 2.

4. In a survey of 320 persons, number of persons taking tea is 210, taking milk is 100 and coffee is 70. Number of persons who take tea and milk is 50, milk and coffee is 30, tea and coffee is 50. The number of persons all three together is 20. Find the number of people who take neither tea nor coffee nor milk.

See Topic: SET THEORY, Long Answer Type Question No. 5.

5. Express  $\begin{bmatrix} -3 & 4 & 1 \\ 2 & 3 & 0 \\ 1 & 4 & 5 \end{bmatrix}$  as a sum of a symmetric and a skew-symmetric matrix.

See Topic: MATRICES, Short Answer Type Question No. 6.

6. If  $\alpha, \beta, \gamma$  be the roots of the equation  $x^3 + 2x^2 + 3x + 4 = 0$ , then find the equation whose roots are  $1 + \frac{1}{\alpha}$ ,  $1 + \frac{1}{\beta}$  and  $1 + \frac{1}{\gamma}$ .

See Topic: POLYNOMIAL, Short Answer Type Question No. 14.

Group - C

(Long Answer Type Questions)

7. a) Verify whether the matrix  $\begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$  is orthogonal.

b) Solve the following system of linear equations by using Cramer's Rule:

$$2x + 5y + 3z = 9$$

$$3x + y + 2z = 3$$

$$x + 2y - z = 6$$

c) If  $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 1 \\ 2 & -2 \\ 3 & 3 \end{bmatrix}$ , find  $AB$ .

d) Show that  $\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \sin \theta \end{bmatrix} \times \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \sin \theta \end{bmatrix} = \begin{bmatrix} \cos 2\theta & -\sin \theta(\sin \theta + \cos \theta) \\ \sin \theta(\sin \theta + \cos \theta) & 0 \end{bmatrix}$

a) See Topic: MATRICES, Short Answer Type Question No. 4.

b) See Topic: MATRICES, Long Answer Type Question No. 10(a).

c) See Topic: MATRICES, Short Answer Type Question No. 3.

d) See Topic: MATRICES, Long Answer Type Question No. 10(b).

8. a) Evaluate any two:

i)  $\lim_{x \rightarrow 0} \frac{\tan 2x - x}{3x - \sin x}$

ii)  $\lim_{x \rightarrow 0} \frac{x \log \sqrt{1+x}}{\sin^2 x}$

iii)  $\lim_{x \rightarrow a} \frac{1 - \cos(x-a)}{(x-a)^2}$

b) Evaluate  $\int_0^{\pi/2} x^2 \sin x \, dx$

c) Differentiate  $\frac{x^3}{(1+x^3)}$  with respect to  $x^4$ .

a) See Topic: **LIMIT, CONTINUITY & DIFFERENTIABILITY**, Long Answer Type Question No. 8.

b) See Topic: **DEFINITE INTEGRALS**, Short Answer Type Question No. 4.

c) See Topic: **SUCCESSIVE DIFFERENTIATION**, Short Answer Type Question No. 4.

9. a) If  $A = \{a, b, c, d, e\}$ ,  $B = \{c, a, e, g\}$  and  $C = \{b, e, f, g\}$ , then show that

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C).$$

b) If  $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ , then find  $A^2$  and show that  $A^2 = A^{-1}$ .

c) Find the maxima and minima of  $x^3 - 6x^2 + 9x - 8$ .

a) See Topic: **SET THEORY**, Long Answer Type Question No. 6.

b) See Topic: **MATRICES**, Long Answer Type Question No. 3.

c) See Topic: **MISCELLANEOUS**, Short Answer Type Question No. 1.

10. a) Determine whether the function

$$f(x, y) = \frac{xy}{x^2 + y^2} \text{ if } (x, y) \neq (0, 0) \\ = 0 \text{ if } (x, y) = (0, 0)$$

is continuous at the origin.

b) Apply Descartes' rule of signs to find the nature of roots of the equation  $x^4 + 2x^2 + x - 12 = 0$

c) State Cauchy's mean value theorem.

a) See Topic: **LIMIT, CONTINUITY & DIFFERENTIABILITY**, Long Answer Type Question No. 9.

b) See Topic: **POLYNOMIAL**, Long Answer Type Question No. 9.

c) See Topic: **EXPANSION OF FUNCTION**, Short Answer Type Question No. 5.

11. a) Find the value of 'a' and 'b' for which the system of equations

$$x + 2y + z = 1$$

$$2x + y + 3z = b$$

$$x + ay + 3z = b + 1$$

has (i) unique solution, (ii) many solutions.

b) Solve the following system of equations by matrix inversion method

$$x + y + z = 6$$

$$x - 2y + z = 0$$

$$2x - y + z = 3$$

c) Find out the rank of the matrix  $\begin{bmatrix} 2 & -4 & 6 \\ 2 & 3 & -1 \\ 3 & 1 & 2 \end{bmatrix}$

See Topic: MATRICES, Long Answer Type Question No. 11.

12. a) If  $u = \cos^{-1} \left\{ \frac{x+y}{\sqrt{x} + \sqrt{y}} \right\}$ , then show that  $x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} + \frac{1}{2} \cot u = 0$

b) If  $PSQ$  be a focal chord of a conic with focus  $S$  and semi latus rectum  $L$ , then prove that  $\frac{1}{SP} + \frac{2}{SQ} = \frac{2}{L}$ .

c) Find the point on the conic  $6/r = 1 + 4 \cos \theta$  whose vertical angle is  $\pi/3$ .

a) See Topic: SUCCESSIVE DIFFERENTIATION, Long Answer Type Question No. 3.

b) & c) See Topic: POLAR EQUATIONS, Long Answer Type Question No. 1(a) & (b).